HR - Automatic Concept Formation in Finite Algebras

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We are investigating how and why mathematicians invent new concepts while developing a theory, and implementing our ideas into the HR system, which automatically produces, assesses and displays concepts in finite algebras, such as finite group theory. We first determined a reason for HR to produce concepts - to classify a given set of groups up to isomorphism. Doing so would involve inventing concepts which help describe groups, so a classification can occur, and inventing concepts which help generate new examples of groups, so that improvements to the classification are necessitated, perpetuating the process. Next, we developed measures to tell us how interesting the concepts produced were (see Colton 1997). This helped us determine the kinds of concepts HR should produce and with this in mind, we implemented production rules taking one (or two) concepts as input and outputting a new concept.

Computational Model

There are three levels of processing which allow HR to operate. In the interests of good methodology, we make clear distinctions between these levels. Firstly, HR uses production rules to transform an old concept (or two old concepts) into a new one. For example, given the group operation: $a*b=c$, then the 'match' production rule invents concepts such as $a*a=a$, where some or all of the parameters are equal.

A second layer of processing takes a newly invented concept and assesses its worth in terms of how well it classifies groups. To do this, we think of each concept as a function taking a single group as input, with the output describing the input group, eg. above, the function would be $f(G)=\{a \in G : a*a=a\}$. To determine the concept's value, HR compares how the concept describes pairs of group tables. If two group tables are isomorphic, then they should have the same description. If they are not isomorphic, then ideally they should have different descriptions. HR can count the number of pairs which are described correctly, and this is used to measure the classifying ability of the concept. If exploration, rather than classification, is the aim, an alternative measure is the novelty of a concept: by describing groups, a concept categorises them, and the novelty of this categorisation can be measured.

The top layer of processing keeps an agenda of which old concepts to base new concepts on, and which production rules to use to do this. This agenda is organised using the assessment of the concepts given by the measurements, and so relies on the heuristic that interesting concepts lead to further interesting concepts.

Initial Results and Future Work

Given two isomorphic group tables for the 8 groups of order less than seven, HR invents a concept in seconds which classifies them (ie. for the input group tables, it has different outputs for non-isomorphic tables, but the same output for isomorphic tables). It identifies this concept as being the most interesting so far, and writes and marks up a description of the concepts as a LaTeX script which is displayed for the user. Initial findings suggest that the heuristic of building on the most interesting concepts first is effective.

At present, HR cannot generate new group tables. Using constraint satisfaction technology, future versions of HR will turn the descriptions of the groups into constraints on a search for more group tables. This will involve deriving different types of concepts, such as sequences of groups (eg. central series) and extensions (eg. central extensions), from the descriptive concepts (in this case, the centre of a group). A more detailed description of HR is due to appear in (Bundy 1998).

References

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